doubleeffect

Credit Valuation Adjustment

A credit or market risk?

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Bert-Jan Nauta

Outline

Introduction	Problem definition	Approach	Preliminary results	Conclusions
▪ What is the Credit Valuation Adjustment (CVA) ▪ CVA in Basel 3	▪ Formulation of the problem ▪ Motivation: why is this problem of interest	▪ Potential Approaches ▪ Credit and Market Risk for CVA	▪ Merton model ▪ Hazard Rate model ▪ Migration Matrix model	▪ Models give different results ▪ Interesting questions remain!

Credit Valuation Adjustement (CVA)

- Denote the fair value of a derivative without default risk of both parties in the transaction by V[default-free].
- Denote the fair value of a derivative with default risk of the counterparty by V[cpty risky].
- Denote the fair value of a derivative with default risk of both parties by V[both risky].
- Then CVA and DVA (Debt Value Adjustment) can be defined by:

$$V[\text{cpty risky}] = V[\text{default-free}] - CVA$$

$$V[\text{both risky}] = V[\text{default-free}] - CVA + DVA$$

Credit Valuation Adjustment (CVA)

CVA can be calculated for a non-collateralized transaction by (approximations of):

$$CVA = LGD \sum_{i=0}^{N} P(0,T_i) E[\max(V(T_i),0)\, I_{T_i<\tau<T_{i+1}}]$$

LGD: Loss Given Default as percentage (assumed to be constant)

$P(0,T_i)$: Value of default-free zero coupon bond maturing at T_i

$E[.]$: Expectation value under pricing measure

$V(T_i)$: Value of transaction without default risk

τ: time of default of the counterparty

Credit Valuation Adjustment (CVA)

When the default time and the value are independent, the CVA may be expressed as:

$$CVA = LGD \sum_{i=0}^{N} P(0,T_i) E[\max(V(T_i),0)](PD(T_{i+1}) - PD(T_i))$$

$PD(T_i)$: probability of default until time T_i

The market implied PD can be inferred from counterparty credit spreads (when available).

Problem description

- In Basel 2/3 the counterparty exposure on a transaction is included in the credit risk capital.
- In Basel 3 the calculation of CVA is required, and in the IMM approach, market risk capital for the risk of changes in the credit spread need to be calculated.

Question: Is there overlap between credit risk and market risk of CVA?

Motivation:

- of interest to banks in discussion with regulators
- of scientific interest as a step towards integrated market and credit risk model

Approach to the problem

- Stylized portfolio: a portfolio with a number of independent zero-coupon bonds (that are valued mark-to-market).

- A generic (model-independent) analysis should be the goal, however this is difficult. Therefore, our first approach is to analyze the problem in explicit models. We choose:
  - Merton model
  - Hazard rate model
  - Migration matrix model

Market Risk and Credit Risk capital

We use the following Basel 2-inspired capital definitions:

› Market Risk Capital

$$\text{capital} = 3\sqrt{10}\, VaR$$

VaR is 99% percentile loss on 1-day time horizon based on market value

› Credit Risk Capital

$$\text{capital} = \text{unexpected loss at 99.9\% and 1yr}$$

Unexpected loss = loss at 99.9% - expected loss

Losses do not include market value changes, only defaults!

Merton model

› The value of the assets is assumed to follow a geometric Brownian motion:

$$dV_t = \mu V_t dt + \sigma V_t dW_t$$

› At time T it is verified if the value of the Assets is larger than the value of the Liabilities:

If $V_T < L$ then company in default

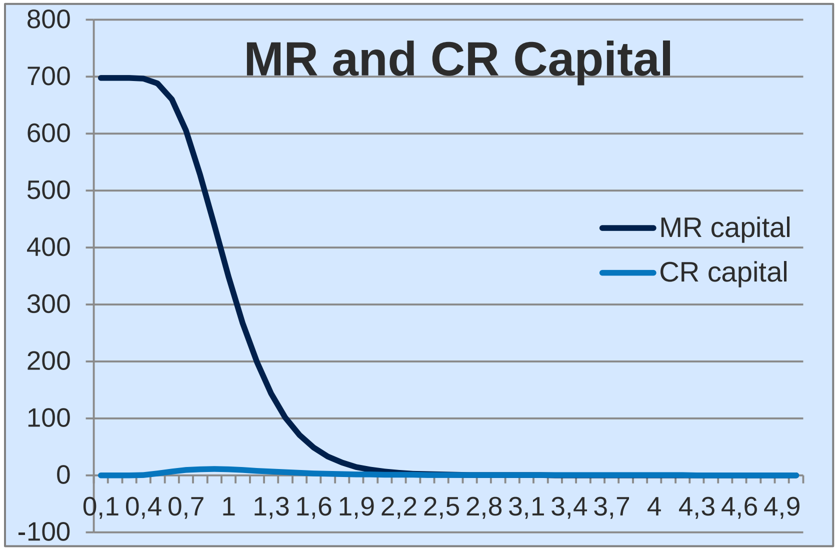
If $V_T \geq L$ then no default

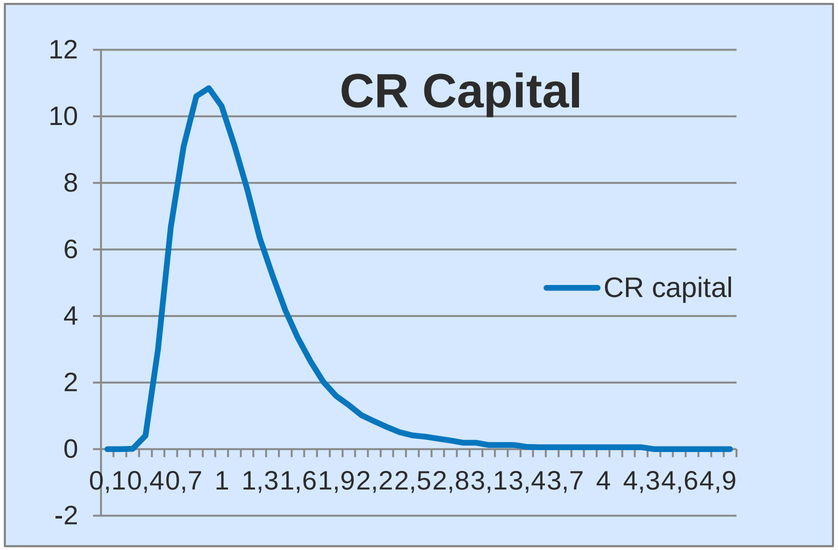
› The pay-off of a zero-coupon (ZC) bond with maturity T and notional 1 is:

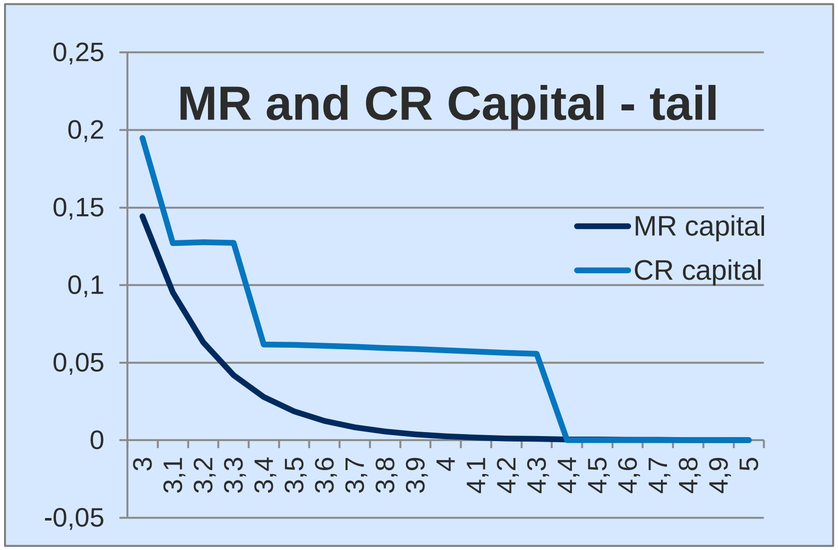
$$Pay - off = \min(1, \frac{V_T}{L})$$

- This defaultable ZC bond consists of a default-free ZC bond plus a short put on the assets.
- Black-Scholes type of valuation is applied.

Market Risk and Credit Risk capital vs Asset Value for 1,000 independent 1yr ZC bonds in Merton model







Assumptions

- Market Risk based on VaR
- Credit Risk based on unexpected loss (at 1yr horizon and 99.9%)
- Interest rates are constant → Market risk capital contains spread risk only
- Asset volatility = 30%, discount factor = 1
- Credit Risk is calculated under risk-neutral measure

Observations from the Merton model

- Market risk capital is high when $V_0 \ll L$, whereas credit risk capital is low.
  - The reason is that although $PD \approx 1$ the $LGD \sim V_0$, which results in the large market risk.
  - And since $PD \approx 1$ the loss at 99.9% is almost equal to the expected loss, resulting in a small unexpected loss and therefore a small credit risk capital.

- MR capital is much higher than CR capital when $V_0 < 3L$, at and beyond $V_0 = 3L$ they are of the same order of magnitude.
  - The PD corresponding to $V_0 = 3L$ is PD= 0.02% (corresponding to AAA/AA).

Hazard rate model

› In the hazard rate model the default time, τ, is assumed to follow the rule:

$$Q(\tau > T) = e^{-\int_0^T \gamma(u)du}$$

γ: hazard rate function

› Value of a defaultable ZC bond:

$$V = P(0,T)[1 - LGD\left(1 - e^{-\int_0^T \gamma(u)du}\right)]$$

$P(0,T)$: value of default-free ZC bond

LGD: Loss Given Default (assumed to be constant)

Properties of the hazard rate model

- The PD can be expressed as:

$$\mathrm{PD} = 1 - e^{-\int_0^T \gamma(u)du}$$

- The LGD is not implied (in contrast to Merton model) and can be chosen as a constant.

- The credit spread can be defined as:

$$S = \left(\frac{1}{T}\right)\ln\left(\frac{P(0,T)}{V}\right) = -\left(\frac{1}{T}\right)\ln(1 - LGD(1 - e^{-\overline{\gamma}T}))$$

with $\overline{\gamma} = (\frac{1}{\mathrm{T}})\int_0^T \gamma(u)du$

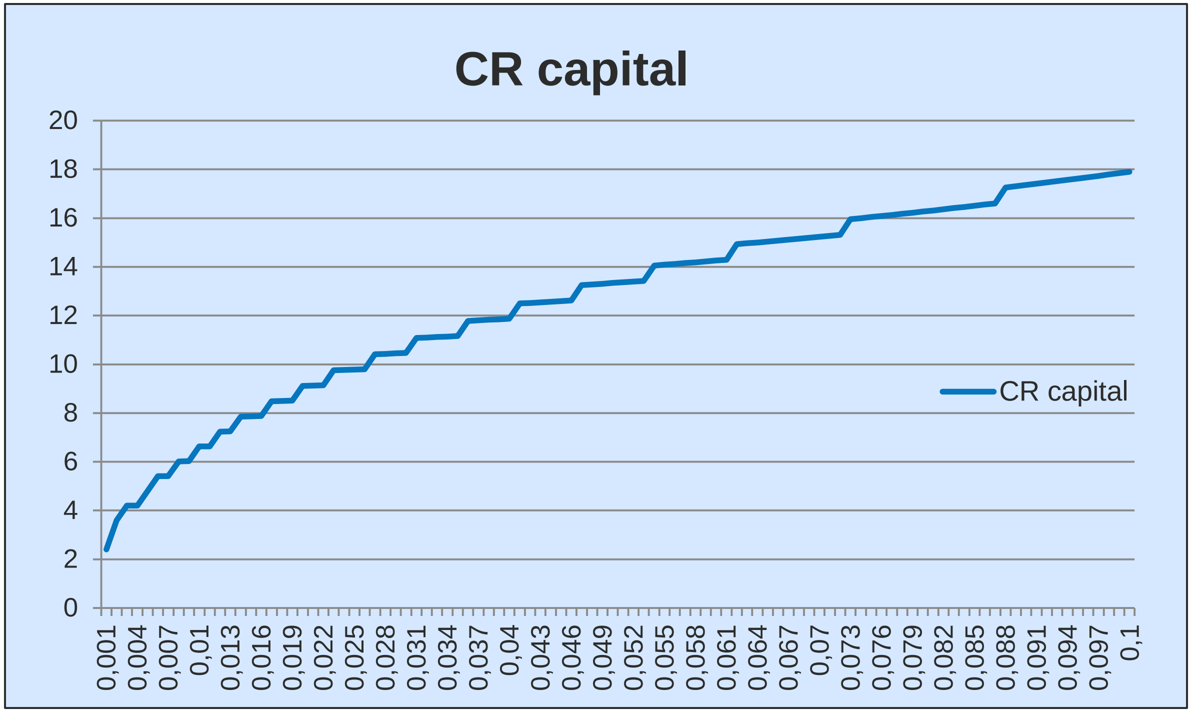
The credit spread is non-stochastic!

Market Risk and Credit Risk capital for 1,000 independent 1yr ZC bonds in Hazard rate model

› MR capital: the credit spread is non-stochastic, hence (approximately):

MR capital = 0

› CR capital is determined by PD and LGD = 40%.



Migration matrix model

An example of a 1-year migration matrix, Standard and Poor's 1991-2001, taken from Schonbucher 2003.

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	89.10	9.63	0.78	0.19	0.30	0	0	0
AA	0.86	90.10	7.47	0.99	0.29	0.29	0	0
A	0.09	2.91	88.94	6.49	1.01	0.45	0	0.09
BBB	0.06	0.43	6.56	84.27	6.44	1.60	0.18	0.45
BB	0.04	0.22	0.79	7.19	77.64	10.43	1.27	2.41
B	0	0.19	0.31	0.66	5.17	82.46	4.35	6.85
CCC	0	0	1.16	1.16	2.03	7.54	64.93	23.19
D	0	0	0	0	0	0	0	100

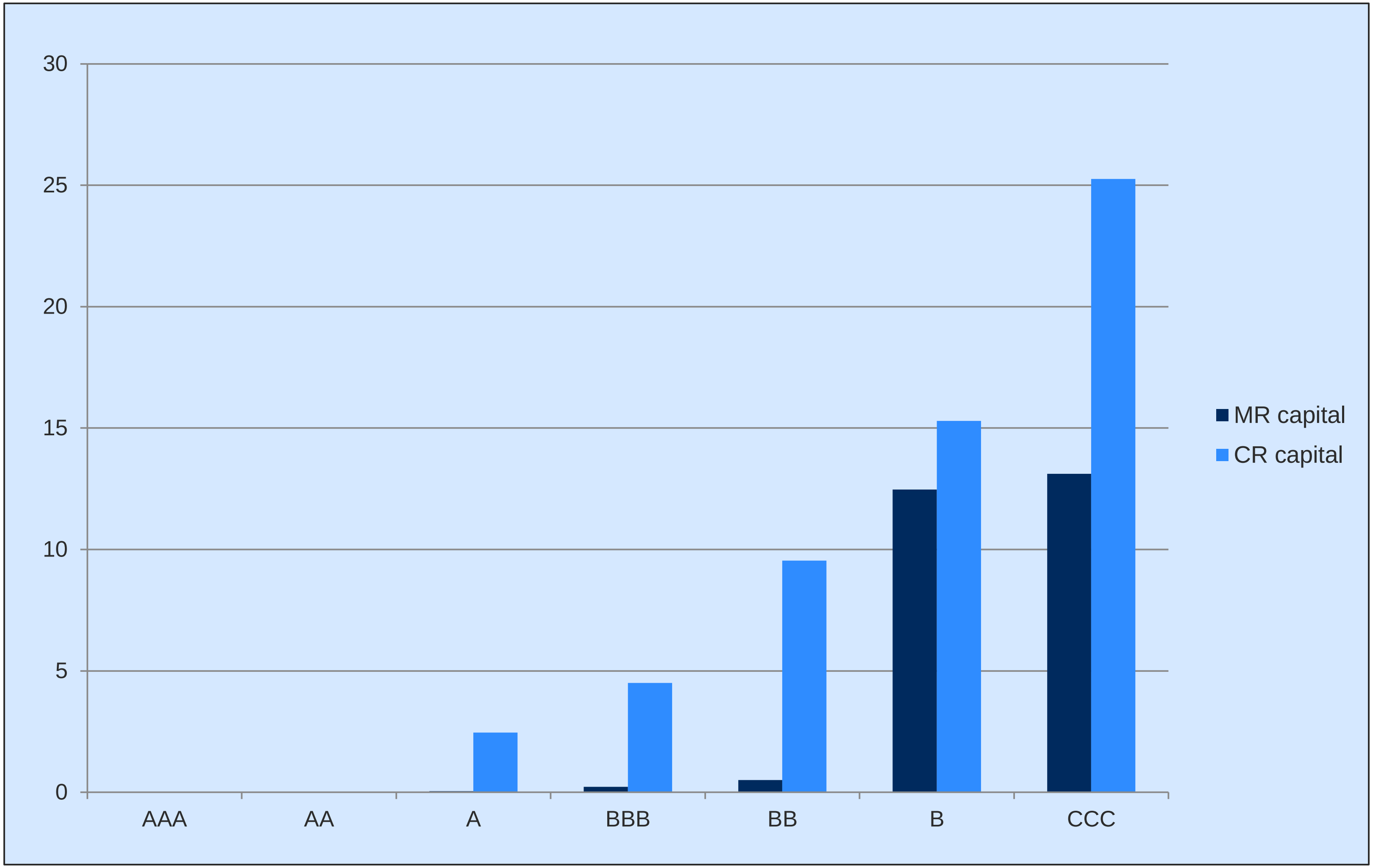
Capital calculations in the Migration Matrix model

- Credit Risk capital calculation based on PD.

- Market Risk capital calculation - steps and assumptions:
  1. Assign to each rating a spread based on PD
  2. Under assumption of Markov property* calculate the generator
  3. From the generator determine the 1-day migration matrix
  4. From the 1-day migration matrix and the spreads determine the 1-day P&L distribution assuming independence of the 1,000 ZC bonds
  5. Take 99% Loss to find the VaR and scale to get the MR capital

* while working for ABN AMRO, T Mexner and BJN developed a tractable extension to non-Markovian evolutions.

Market Risk and Credit Risk capital for 1,000 independent 1yr ZC bonds in Migration Matrix Model



Conclusions

- A consistent treatment of the market and credit risk in the CVA of traded products is an important step towards a combined market and credit risk model.
- The basic models considered here, give very different credit and market risk capital profiles.
- This question provides an interesting and challenging subject for an ambitious student.

Bright ideas ...

... are needed!